Lecture 1: Consumption Based Model

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Introduction (1)

- The central idea of modern finance is that prices are generated by expected discounted payoffs,

$$p_t = E_t(m_{t+1}x_{t+1})$$

where p_t = asset price, x_{t+1} = asset payoff, m_{t+1}= stochastic discount factor

- Examples:

Asset	Price	Payoff
Stock	$\mathcal{S}_t$	$\mathcal{S}_{t+1} + \mathcal{D}_{t+1}$
Return	1	R_{t+1}
Option	$\mathcal{C}$	$\max\{\mathcal{S}_T - K, 0\}$

Introduction (2)

- Different models imply different m_{t+1}
- Models: consumption-based model, CAPM, ICAPM, APT, option pricing, etc...
- We start with the most general model built by Debreu, Arrow, Lucas and Prescott and then proceed to the specializations of it
- What theories match the facts?
  - Each model leads to predictions stated in terms of returns, price-dividend ratios, expected return-beta representations, moment conditions, etc...
  - Theories should be frequently assessed and modified in view of new evidence

Overview

- Some basic facts
- Study the asset pricing implications of household portfolio choice
- Consider the quantitative implications of a second-order approximation to asset return equations
- Reference: Mehra and Prescott (JME, 1984)

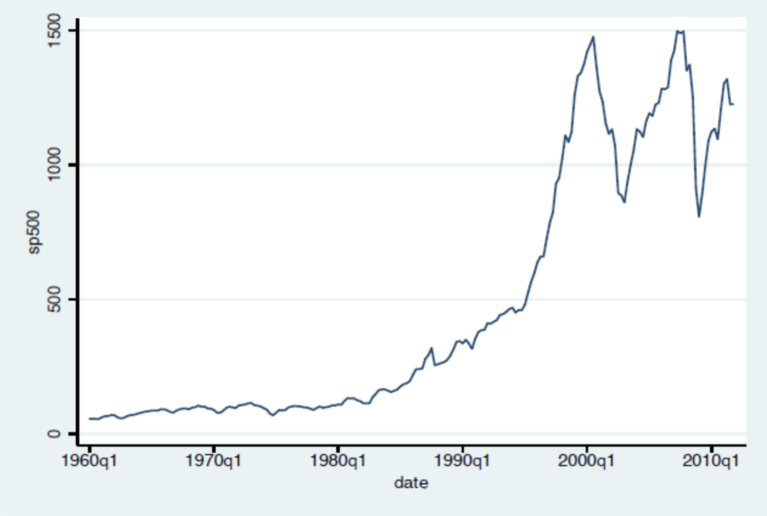
Some Facts (1)

- Stock returns:
  - Average real return on SP500 is 8% per year
  - Stock returns are very volatile: $\sigma(R) = 17\%$ per year
  - Stock returns show very little serial correlation ($\rho = 0.08$ quarterly data, -0.04 annual data)
- Bond returns:
  - The average risk free rate is 1% per year (US Tbill - Inflation)
  - The risk free rate is not very volatile: $\sigma(R) = 2\%$ per year but is persistent ($\rho = 0.6$ in annual data) leading to medium-run variation
- These imply that the equity premium is large 7% per year on an annual basis

Some Facts (2)

- Return-Forecasting Regressions
  - The regression of returns on the dividend yield gives important insights:
  - The coefficient estimate is large, 1 percentage point increase in dividend yield forecasts almost four percentage point higher return.
  - Prices rise by an additional 3 percentage points.
  - The percentage point variation in expected returns is large, almost equal to the equity premium
  - The slope coefficients and R^2 rise with horizon
- High prices, relative to dividends, have preceded many years of poor returns. Low prices have preceded high returns.

S&P 500 (1)



1500
1000
500
0
sp500
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
date

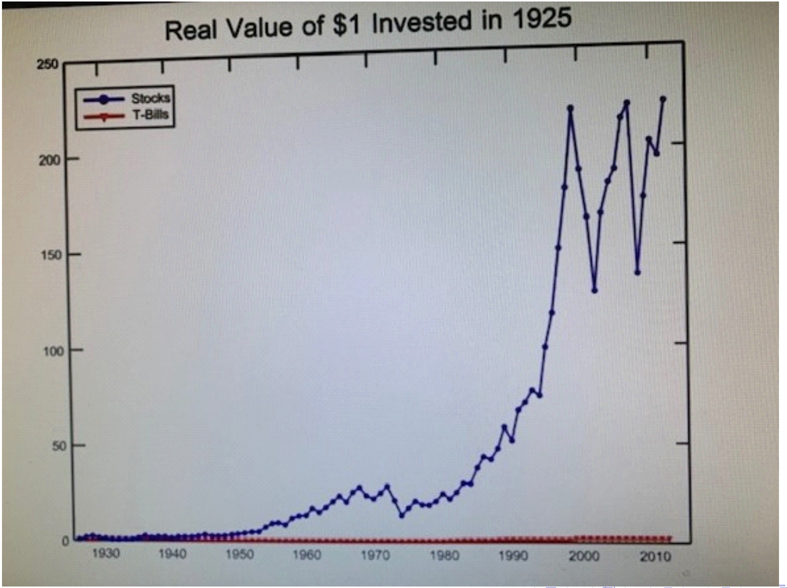
S&P 500 (2)


FRED
S&P 500
4,800
4,400
4,000
3,600
3,200
2,800
2,400
2,000
1,600
1,200
Index
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
Source: S&P Dow Jones Indices LLC
fred.stlouisfed.org

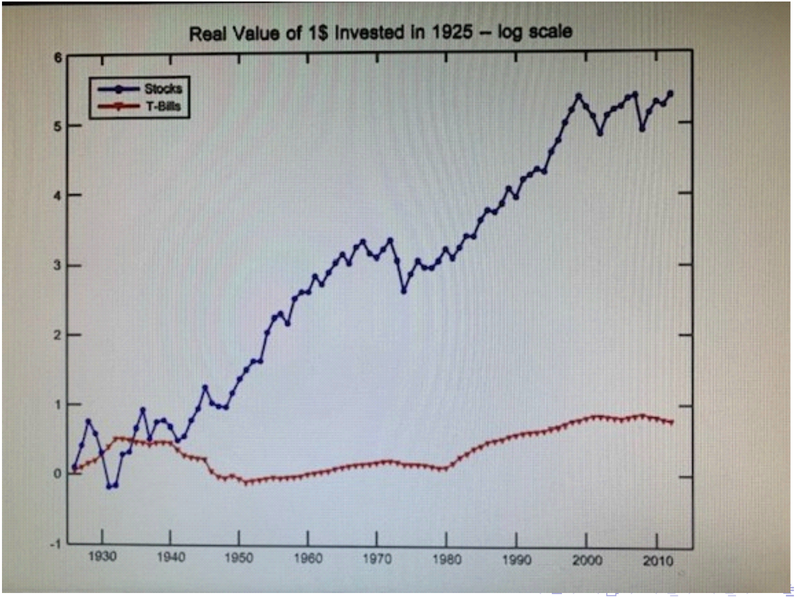
Nasdaq

FRED
NASDAQ Composite Index
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
Index Feb 5, 1971=100
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Source: NASDAQ OMX Group
fred.stlouisfed.org

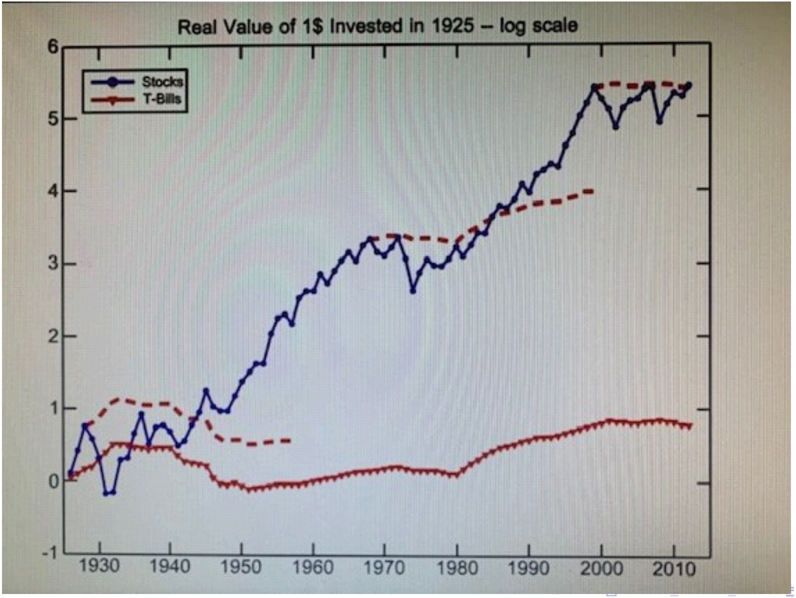
Real value of $1 invested in 1925



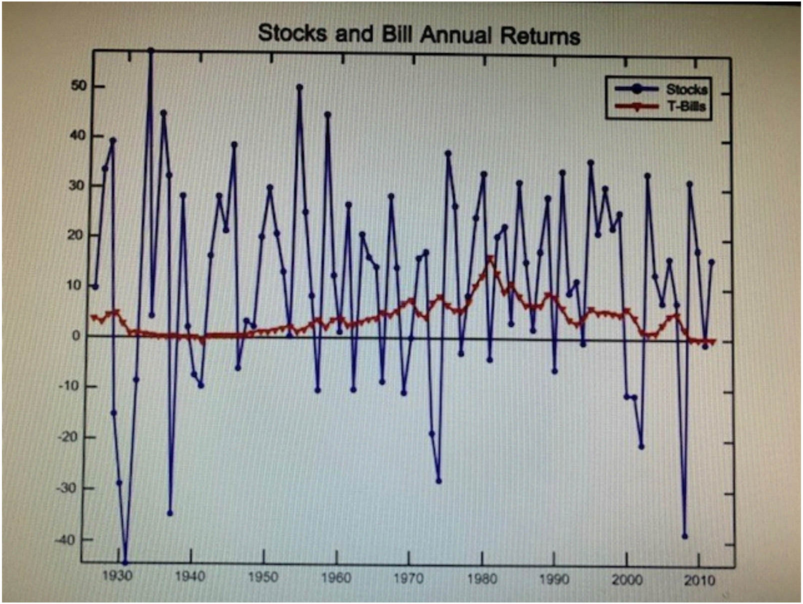
Real value of $1 invested in 1925-log scale



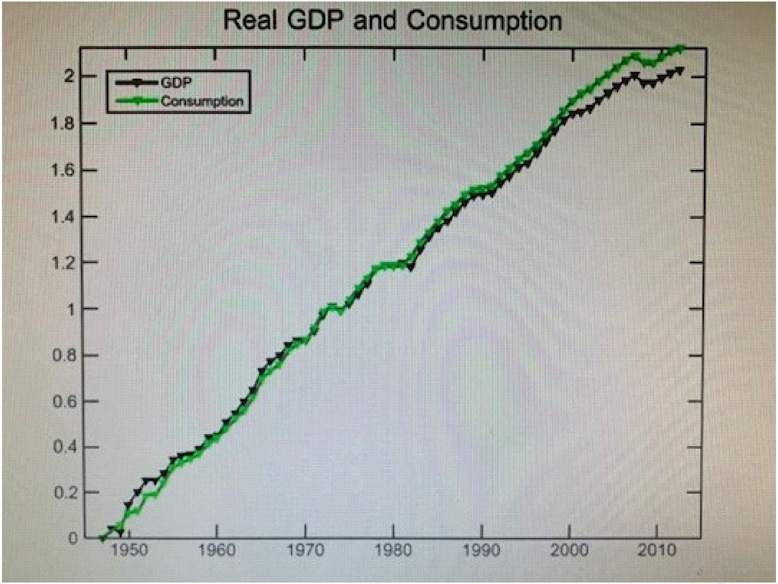
Shifting to bonds



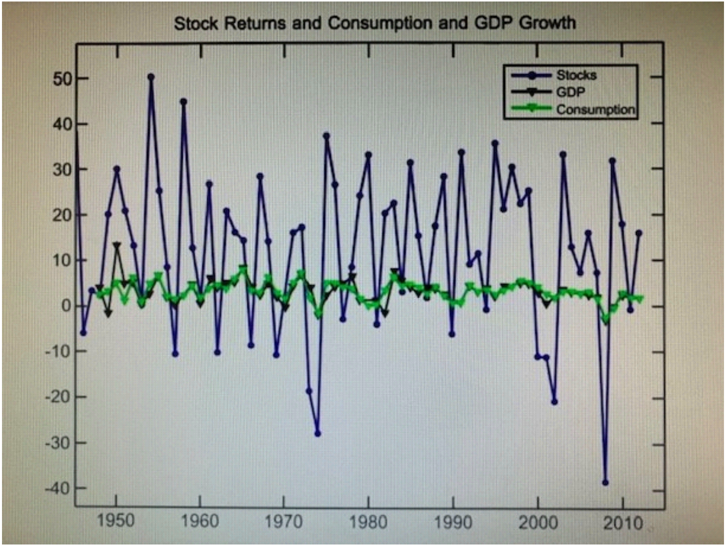
Annual returns



Real GDP and Consumption



Comparison of volatilities



Equity premium and risk

Equity Premium and Risk (all variables measured in real terms)

	Stock Returns	Bond Returns	Stock-Bond	GDP	Consumption
E	8.6	1.3	7.4	3.2	3.3
Stand. Dev	17.6	2.6	18.1	2.6	2.1
Corr	0.99	–0.03	1.00	0.32	0.39

Regression of returns on lagged returns

Regression of returns on lagged returns

Annual data 1927-2008

$$R_{t+1} = a + bR_t + \varepsilon_{t+1}$$

	b	t(b)	R^2	E(R)	$\sigma(E_t(R_{t+1}))$
Stock	0.04	0.33	0.002	11.4	0.77
T bill	0.91	19.5	0.83	4.1	3.12
Excess	0.04	0.39	0.00	7.25	0.91

Return-Forecasting Regression

Table I

Return-Forecasting Regressions

The regression equation is $R^e_{t\to t+k} = a + b \times D_t/P_t + \varepsilon_{t+k}$. The dependent variable $R^e_{t\to t+k}$ is the CRSP value-weighted return less the 3-month Treasury bill return. Data are annual, 1947–2009. The 5-year regression t-statistic uses the Hansen–Hodrick (1980) correction. $\sigma[E_t(R^e)]$ represents the standard deviation of the fitted value, $\sigma(\hat{b} \times D_t/P_t)$.

Horizon k	b	$t(b)$	R^2	$\sigma[E_t(R^e)]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

Return predictability SP 500 & SP 500 weighted

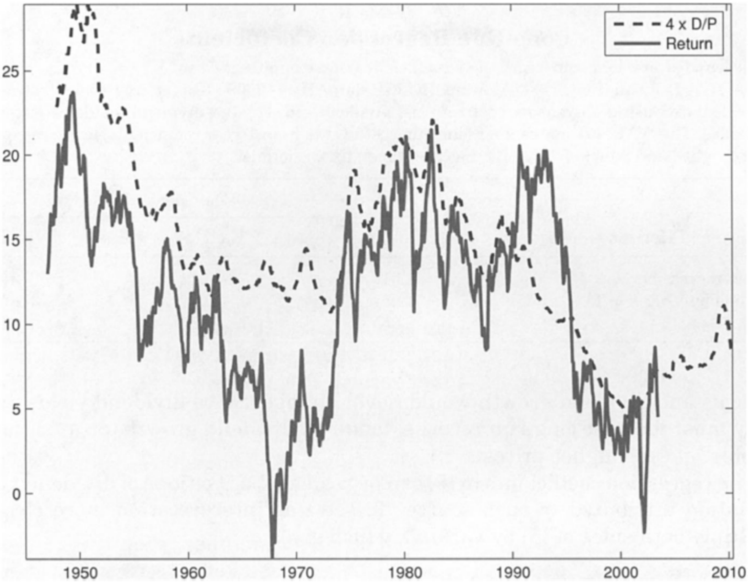
- Cambpell and Shiller (and many others) consider the following regression:

$$R^e_{t,t+k} = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_t$$

where $R^e_{t,t+k}$ is the realized cumulative return over k periods.

k	1y	2y	3y	4y	1y	2y	3y	4y
β	3.83	7.42	11.57	15.81	3.39	6.44	9.99	13.54
tstat	2.47	3.13	4.04	4.35	2.18	2.74	3.58	3.83
R^2	0.07	0.11	0.18	0.20	0.06	0.09	0.15	0.17

Return predictability (7-year return)



Excess returns and dividend regressions

OLS Regressions of Excess Returns (value-weighted NYSE—Treasury bill) and Real Dividend Growth on the Value-Weighted NYSE Dividend-Price Ratio

Horizon k (years)	$R^e_{t\to t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$			$\frac{D_{t+k}}{D_t} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$		
	b	$t(b)$	R^2	b	$t(b)$	R^2
1	4.0	2.7	0.08	0.07	0.06	0.0001
2	7.9	3.0	0.12	−0.42	−0.22	0.0010
3	12.6	3.0	0.20	0.16	0.13	0.0001
5	20.6	2.6	0.22	2.42	1.11	0.0200

Sample 1927–2005, annual data. $R^e_{t\to t+k}$ denotes the total excess return from time t to time $t+k$. Standard errors use GMM (Hansen–Hodrick) to correct for heteroskedasticity and serial correlation.

Cross-sectional evidence

- Small firms have high returns on average (size premium)
- Firms with low Tobins' Q (low book/market) have higher returns on average (value premium)
- Firms with high recent returns tend to have high returns in near future (momentum anomaly)

Lucas Representative Agent Economy

- Lucas, Robert E. Jr, 1978, "Asset Prices in An Exchange Economy", Econometrica
- Time: $t = 0, 1, ...\infty$
- Consumers and one Firm
- *Preferences:*

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

- The firm pays dividend $\mathcal{D}_t$ and the price of stock is $\mathcal{S}_t$
- The consumers issue zero coupon bonds

Model

- Let $\mathcal{B}_{j,t}$ price at t of the zero coupon that matures at $t + j$, $\mathcal{B}_{0,t} = 1$
- There are T maturities of debt
- Denote by $B_{j,t}$ and S_t the holdings of j bonds and shares at the start of t

Endowment

- The dividend, $\mathcal{D}_t$, paid start of period t, is exogenous and equal to y_t (the endowment)
- The endowment y_t follows some stochastic process

Budget Constraint

Budget Constraint of the representative consumer:

$$\sum_{j=1}^{T} B_{j,t+1}\mathcal{B}_{j,t} + S_{t+1}\mathcal{S}_t + c_t = \sum_{j=1}^{T} B_{j,t}\mathcal{B}_{j-1,t} + S_t\left(\mathcal{D}_t + \mathcal{S}_t\right)$$

Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) +$$

$$E_0 \sum_{t=0}^{\infty} \lambda_t \left\{ \sum_{j=1}^{T} B_{j,t} \mathcal{B}_{j-1,t} + S_t \left(\mathcal{D}_t + \mathcal{S}_t \right) - \sum_{j=1}^{T} B_{j,t+1} \mathcal{B}_{j,t} - S_{t+1} \mathcal{S}_t - c_t \right\}$$

First Order Conditions:

$$\beta^t u'(c_t) - \lambda_t = 0$$

$$E_t \left[\lambda_{t+1} \left(\mathcal{D}_{t+1} + \mathcal{S}_{t+1}\right)\right] - \lambda_t \mathcal{S}_t = 0$$

for all t

$$E_t \left[\lambda_{t+1} \mathcal{B}_{j-1,t+1}\right] - \lambda_t \mathcal{B}_{j,t} = 0$$

for $j = 1, ..., T$ all t

First Order Conditions for bonds and shares:

$$u'(c_t)\mathcal{S}_t = \beta E_t \left[u'(c_{t+1}) \left(\mathcal{D}_{t+1} + \mathcal{S}_{t+1} \right) \right]$$

for all t

$$u'(c_t)\mathcal{B}_{j,t} = \beta E_t \left[u'(c_{t+1})\mathcal{B}_{j-1,t+1} \right]$$

for $j = 1, ..., T$ all t

Pricing

By replacing back this expression

$$\mathcal{S}_t = E_t \left\{ \sum_{s=1}^{\infty} \beta^s \frac{u'(c_{t+s})}{u'(c_t)} \mathcal{D}_{t+s} \right\} + \lim_{J \to \infty} \beta^J E_t \frac{u'(c_{t+J})}{u'(c_t)} \mathcal{S}_{t+J}$$

$$\mathcal{B}_{j,t} = E_t \left\{ \beta^j \frac{u'(c_{t+j})}{u'(c_t)} \mathcal{B}_{j-1,t+1} \right\}, \text{ for } j = 1, ..T$$

Stochastic Discount Factor

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

Asset payoff

$$x_{t+1} = \mathcal{D}_{t+1} + \mathcal{S}_{t+1},$$

for stock

$$x_{t+1} = \mathcal{B}_{j-1,t+1},$$

for a j maturity bond
Price

$$\mathcal{S}_t \text{ for stock}$$

$$\mathcal{B}_{j,t}, \text{ for a } j \text{ maturity bond}$$

Price

$$p_t = E_t m_{t+1} x_{t+1}$$

$$\mathcal{S}_t = E_t \left\{ \beta \frac{u'(c_{t+1})}{u'(c_t)} \left(\mathcal{D}_{t+1} + \mathcal{S}_{t+1} \right) \right\}$$

$$\mathcal{B}_{j,t} = E_t \left\{ \beta \frac{u'(c_{t+1})}{u'(c_t)} \mathcal{B}_{j-1,t+1} \right\}$$

Competitive Equilibrium

Allocation and prices such that (i) the allocation solves the representative household problem of maximizing his intertemporal utility given his budget constraint and (ii) prices clear the markets.

Equilibrium

Total supply of the good

$$y_t = \mathcal{D}_t, \text{ all } t$$

Markets

$$c_t = y_t, \text{ all } t$$

$$S_t = 1, \text{ all } t$$

$$B_{j,t} = 0, \text{ all } j \text{ and } t$$

Equilibrium

- replace $c_t = y_t$ in and solve for prices $\mathcal{B}_{j,t}$ and $\mathcal{S}_t$